XII MATHS

1. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ & $\frac{\pi}{2}$ with y & z axes respectively. $3(\pm i + j)$.

Q.2 Find a vector of magnitude 5 units & ⊥ to the resultant of the vectors $2\hat{i}+3\hat{j}-\hat{k}$ & $\hat{i}-2\hat{j}+\hat{k}$. $\pm \frac{5}{\sqrt{10}}(3\hat{i}+\hat{j})$.

Q.3 If the vectors $\hat{j}+\hat{k}$ & $3\hat{i}-\hat{j}+4\hat{k}$ represent two side vector $\vec{AB}$ & $\vec{AC}$ respectively of ΔABC. Find the length of median through A. $5\sqrt{2}$.

Q.4 Find $\vec{a}+\vec{b}$. If the points A(2, a, 3) B(3, -5, b) & C(-1, 11, 9) are collinear. 0

Q.6 P & Q are points with P.V. $3\vec{a}-2\vec{b}$ & $\vec{a}+\vec{b}$ respt. write the P.V. of R which divides the line segment PQ externally in 2:1. $-\vec{a}+4\vec{b}$.

Q.7 If $\vec{a}$ & $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ $|\vec{a}|=5$ Find $|\vec{b}|$ (12)

Q.8 If $\vec{a}$ & $\vec{b}$ are unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle b/w $\vec{a}$ & $\vec{b}$. 120°

Q.9 If $\vec{a}=4\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=5\hat{i}+2\hat{j}-3\hat{k}$, find the angle b/w the vectors $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$. $\cos^{-1}\left(\frac{-17}{\sqrt{565}}\right)$.

Q.10 If $\vec{p}=5\hat{i}+\lambda\hat{j}-3\hat{k}$ & $\vec{q}=\hat{i}+3\hat{j}-5\hat{k}$, then find the values of λ so that $\vec{p}+\vec{q}$ & $\vec{p}-\vec{q}$ are perpendicular vectors. ± 1.

Q.11 Find λ, when the scalar projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units. (5)

Q.12 If $\vec{a}$, $\vec{b}$ are two non zero, non collinear vectors such that $|\vec{a}+\vec{b}|=|\vec{a}|$, then prove that $2\vec{a}+\vec{b}$ is ⊥ to $\vec{b}$.

Q.13 Prove that $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

Q.14 $\vec{a}=3\hat{i}-\hat{j}$, $\vec{b}=2\hat{i}+\hat{j}-3\hat{k}$, express $\vec{b}$ as $\vec{b_1}+\vec{b_2}$ where $\vec{b_1} \parallel \vec{a}$ & $\vec{b_2} \perp \vec{a}$. $\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}$ & $\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

P.T.O.

(15) If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, find the angle b/w $\vec{a}$ & $\vec{b}$. 60°

(16) If $\hat{a}, \hat{b}, \hat{c}$ are mutually perpendicular unit vectors, find $|2\hat{a}+\hat{b}+\hat{c}|$ $\sqrt{6}$

(17) If $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitude 3, 4, 5 units respectively, if each of these is $\perp$ to the sum of other two vectors, find $|\vec{a}+\vec{b}+\vec{c}|$ (5√2)

(18) Dot product of a vector with $3\hat{i}-5\hat{k}$, $2\hat{i}+7\hat{j}$ & $\hat{i}+\hat{j}+\hat{k}$ are respectively $-1, 6, 5$, find the vector $3\hat{i}+2\hat{k}$

(19) $|\vec{a}|=3$, $|\vec{b}|=\frac{\sqrt{2}}{3}$, $\vec{a}\times\vec{b}$ is a unit vector, find angle b/w $\vec{a}$ & $\vec{b}$. 45°

(20) Find a vector of magnitude $\sqrt{171}$ which is $\perp$ar to both $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. $\pm(\hat{i}-11\hat{j}-7\hat{k})$

(21) If $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$, find a unit vector $\perp$ar to $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$. $\pm\frac{1}{3}(2\hat{i}-2\hat{j}-\hat{k})$

(22) Find a unit vector $\perp$ to the plane of $\triangle ABC$, where the coordinates of the vertices are $A(3,-1,2)$, $B(1,-1,-3)$ & $C(4,-3,1)$. $\pm\left(\frac{-10\hat{i}-7\hat{j}+4\hat{k}}{\sqrt{165}}\right)$.

(23) If $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$, $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$, find $\vec{d}$, such that $\vec{d}$ is $\perp$ to both $\vec{a}$ & $\vec{b}$ & $\vec{c}\cdot\vec{d}=15$ $\frac{5}{3}(3\hat{i}-\hat{j}-14\hat{k})$

(24) If $\vec{r}=x\hat{i}+4\hat{j}+3\hat{k}$, find $(\vec{r}\times\hat{i})\cdot(\vec{r}\times\hat{j})+xy$. (6)

(25) If $\vec{a}+\vec{b}+\vec{c}=0$ prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

(26) If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$, $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is $\parallel$ to $\vec{b}-\vec{c}$, $\vec{a}\neq\vec{d}$, $\vec{b}\neq\vec{d}$.

(27) If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, $\vec{a}\neq 0$, prove that $\vec{b}=\vec{c}$.

(28) If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$, prove that $\vec{a}\perp\vec{b}\perp\vec{c}$ & $|\vec{b}|=1$, $|\vec{c}|=|\vec{a}|$

P.T.O.

(29) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$, Find $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ & $\vec{a} \cdot \vec{c} = 3$ $\quad \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$.

(30) Using vector Find the area of $\triangle ABC$, where A(1,1,2), B(2,3,5) & C(1,5,5) (5√6/2)

(31) Find λ if $\hat{i} + 3\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} - \hat{k}$, $\lambda\hat{i} + 3\hat{k}$ are coplanar vectors. $\lambda = 7$

(32) If $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, $a \neq b \neq c \neq 1$, prove that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

(33) Find λ if A(−1, 4, −3) B(3, λ, −5), C(−3, 8, −5) & D(−3, 2, 1) are coplanar. (2)

(34) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are coplanar.

(35) If $\vec{a}, \vec{b}, \vec{c}$ are mutually ⊥ vectors of same magnitude, then show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with $\vec{a}, \vec{b}, \vec{c}$ & this inclination is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

---

(36) Find the coordinates of the foot of perpendicular drawn from the point A(1, 8, 4) to the line joining the point B(0, −1, 3) & C(2, −3, −1) also Find the Image of A with respect to line mirror AB. F(−5/3, 2/3, 19/3). I $\left(\frac{-13}{3}, \frac{20}{3}, \frac{26}{3}\right)$.

(37) Find the image of the point P(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, I(1, 0, 7)

(38) Find the Image of (1,3,4) in the plane $2x - y + z = -3$, I(−3, 5, 2)

(39) Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured ∥ to $2x = y = z$ (6)

(40) Find the distance of the point (2, 12, 15) from the point of intersection of the plane $x - 2y + z = 0$ & line $\frac{x-2}{3} = \frac{y+4}{4} = \frac{z-2}{2}$ (13)

(41) Find the distance of the point A(−2, 3, −4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured

P. T. O

(4)

|| to the plane $4x+12y-3z+1=0$ (7½)

(42) Show that $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines & also find the point of intersection $(-1, -1, -1)$

(43) Find the eqn of line passing the point $(-1, 3, -2)$ and ⊥ to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ & $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$

(44) Find the S.D b/w the lines $x+1 = 2y = -12z$ & $x = y+2 = 6z-6$ (2)

(45) Find the S.D. b/w the lines $\vec{r} = \hat{i}+2\hat{j}-4\hat{j}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ & $\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$ $\frac{\sqrt{293}}{7}$

(46) Find the Eqn of Plane passing through the $(1,2,3)$ & $(0,-1,0)$ & || to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

$6x-3y+z-3=0$

(47) Find the Eqn of Plane containing the line $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-4}{-2}$ & the point $(0,6,0)$. $3x+2y+6z+12=0$

(48) Show that $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ & $\frac{x}{1} = \frac{y-7}{3} = \frac{z+7}{2}$ are coplanar. Also find the Eqn of Plane which contains these lines. $2x+y-z=2$

(49) If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ & $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, find value of k, also find the Eqn of Plane which contains these lines. $5x-2y-z=6$

(50) Find the Eqn of Plane passing through the intersection of $x+2y+3z=5$, $3x-2y-z=-1$ & cutting off equal intercepts with x & z axes. $5x+2y+5z-5=0$

P.T.O

(51) Find the Eqn of plane passing through the line of intersection of $\vec{r}\cdot(2\hat{i}+\hat{j}+3\hat{k})=7$ & $\vec{r}\cdot(3\hat{i}-5\hat{j}+3\hat{k})=9$ & the point (2, 1, 3). $\vec{r}\cdot(2\hat{i}-13\hat{j}+2\hat{k})=0$.

(52) Find the Eqn of plane passing through the line of intersection of $2x+3y-z+1=0$ & $x+y-2z+3=0$ & ⊥ to the plane $3x-y-2z-4=0$ $7x+13y+4z-9=0$

(53) Find the Eqn of plane passing through the line of intersection of $2x+y-z=3$ & $5x-3y+4z=9$ & ∥ to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$ $7x+9y+10z=27$

(54) Find the Eqn of planes passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j})-6=0$ & $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at a unit distance from origin. $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})=3$ $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})=-3$

(55) Find the Eqn of the plane determined by the points A(3, −1, 2), B(5, 2, 4), C(−1, −1, 6) & find its distance from (6, 5, 8). $3x-4y+3z-19=0$, $\frac{6}{\sqrt{34}}$

(56) Find the Eqn of plane passing through (1, −1, 2) & ⊥ to each of the planes $2x+3y-2z=5$ & $x+2y-3z=8$. $5x-4y-z-7=0$

(57) Find the Eqn of plane passing through the (2, 1, −1) & (−1, 3, 4) & ⊥ to $x-2y+4z=10$ $18x+17y+4z=41$

(58) Find the coordinates of the point where the line through the points A(3, 4, 1) & B(5, 1, 6) crosses the plane determined by P(2, 1, 2), Q(3, 1, 0) & R(4, −2, 1). $\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$

Probability-

Q.1 A Girl throws a die, If she gets 5 or 6, she Tosses the Coin 3 Times & Noted the Number of heads, If she gets 1, 2, 3 or 4, she tosses a coin two times & noted the number of heads obtained, If she obtained exactly two heads, what is the prob that she threw 1, 2, 3 or 4 with the die. 4/7

Q.2 There are three coins, One is a two headed coin, another is biased coin that comes up head 75% of the time & third is also biased coin that comes up tails 40% of the time, one of three coins is chosen at random & tossed, It shows head, what is the prob that it was two headed coin. 20/47

Q.3 Two Nos are selected at Random from the 1st six positive numbers (Natural). Let X denote the larger of two nos obtained, Find the prob distribution of the X, also find the mean of this distribution.

X :	2	3	4	5	6
P :	1/15	2/15	3/15	4/15	6/15

, mean = $\frac{14}{3}$

Q.4 If A & B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$, $P(A \cap \bar{B}) = \frac{1}{6}$, find P(A) & P(B) 5/6, 4/5

Q.5 Five bad oranges are accidently mixed with 20 good ones, If 4 oranges are drawn One by One with Replacement, then find the Prob Distribution for number of bad Oranges, also find mean & Var. of this dis. Mean = 4/5 Var = 16/25

Q.6 from a lot of 10 bulbs, which includes 3 defective, a sample of 2 bulbs is drawn at random. Find the prob. distribution of the number of defective bulbs.

X :	0	1	2
P :	$\frac{49}{100}$	$\frac{21}{50}$	$\frac{9}{100}$

Q.7 A card from a pack of 52 cards is lost, from the remaining cards of pack, two cards are drawn at random & are found to be both clubs, find the prob of the lost card being of clubs. $\frac{11}{50}$

Q.8 A bag contains 4 balls, two balls are drawn at random & are found to be white, what is the prob that all balls in the bag are white. $\frac{3}{5}$

Q.9 An experiment succeeds thrice as often as it fails, find the prob that in the next 5 trials, there will be at least 3 success. $\frac{459}{512}$

Q.10 How many times must a man toss a fair coin, so that the prob of having at least one Head is more than 80%. (5)

Q.11 A die is thrown again & again until three sixes are obtained find the prob of obtaining the third six in 6th throw. $\frac{625}{23328}$.

Q.12 A can hit a target 4 times in 5 shots, B three times in 4 shots, C Twice in 3 shots, they fire a volley, what is the prob that at least 2 shots hit. $\frac{5}{6}$

Q.13 Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind & cooperative with their patients & so are very popular, while the other two remains reserved. For a health camp, three doctors are selected at random, find the Prob distribution of the No of very popular doctors.

x	1	2	3
p	3/28	15/28	10/28

Q.14 Assume that each born child is equally likely to be a boy or girl, If a family has two children, what is the conditional prob that both are girls, given that (i) The youngest is a girl (ii) at least One is a girl. 1/2, 1/3

Q.15 Two integers are selected at random from 1 to 11, If the sum of integers 1 to 11, If the Sum of Nos chosen is even, find the prob that both Nos are odd. 3/5

Q.16 A bag I contains 5 Red & 4 white balls & bag II contains 3R & 3W balls. Two balls are transfered from bag I to II the one ball is drawn from the bag II, If ball drawn from bag II is Red, the Prob that one ball is red & One white ball is transfered from bag I to II. 20/37

Q.17 For 6 Trials of an experiment, x is the Binomial variate such that 9 P(x=4) = P(x=2), Find the Prob of Success. p=1/4

P.T.O

Q.18 If A & B are independent events associated with a random experiment, then Prove that- $\bar{A}$ & $\bar{B}$ are also independent events.

Q.19 A Student is given a test with 8 Ques of true-false type. If he gets 6 or more correct Ans he declared pass. Given that he guesses the ans to each Question. Find the prob that she/he will pass the Test 37/256

Q.20 A man is known to speak truth 3 out of 4 times. He throw a die & reports that it is a six. Find the Prob that it is actually six. 3/8.

Q.21 A, B, C Tosses a coin Turn by Turn, A First, B Second & C third. If any will get ~~six~~ Head, wins the game. Find their respective winning probabilities. 4/7, 2/7, 1/7.

AREA By INTEGRATION

Q.22. Find the area of region bounded by $y^2 = 4x$ & $4x^2+4y^2=9$. $\frac{2\sqrt{2}}{3} + \frac{9\pi}{8} - \frac{\sqrt{2}}{2} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

Q.23 Prove that the curves $y^2=4x$ & $x^2=4y$ divides the area of square bounded by $x=0$, $x=4$, $y=4$, $y=0$ into three equal parts.

Q.24 Using integration find the area of triangle whose vertices are A(-1, 2), B(1, 5), C(3, 4) (4)

Q.25 Find the area $\left\{(x,y) : \frac{x^2}{9}+\frac{y^2}{4} \le 1 \le \frac{x}{3}+\frac{y}{2}\right\}$

$\frac{3}{2}(\pi-2)$

P.T.O.

Q.26 Find the area of the region bounded by $y=x^2$ & $y=|x|$. 1/3

Q.27 Find area enclosed between $4y=3x^2$ & $3x-2y+12=0$ 27

Q.28 Find the area of Δ formed by (+ve) x axis & Tangent & Normal to the circle $x^2+y^2=4$ at $(1,\sqrt{3})$ $2\sqrt{3}$

Q.29 Using integration find the area of the region bounded by $4x-y+5=0$, $x+y-5=0$, $x-4y+5=0$. 15/2

Q.30 Find the area of ΔABC where $A(2,-2)$, $B(4,3)$ & $C(1,2)$. 13/2

Q.31 Find the area of circle, exterior to the parabola $y^2=6x$. $x^2+y^2=16$ $16\pi-\left[\frac{2\sqrt{3}}{3}+\frac{8\pi}{3}\right]$

(II) Find the area $\{(x,y) : x^2+y^2 \le 2ax\}$

Q.32 (I) Find the area of the region $y^2 \ge 4x$, $x,y \ge 0$ $\frac{(3\pi-8)\cdot a^2}{12}$

$\{(x,y) : \{|x+2| \le y \le \sqrt{20-x^2}\}$ $5\pi-2$

Q.33 Find the area bounded by $y=|x-1|$ & $y=3-|x|$ (4)

Q.34 Area of the region enclosed by $x^2+y^2=1$ & $x^2+(y-1)^2=1$. $(2\pi/3-\sqrt{3}/2)$

Q.35 Find the area of the region in 1st Quadrant enclosed by x axis, $y=x$ & $x^2+y^2=32$ 4π

Solve Differential Eq^{n}s.

Q.36 (I) $e^x \tan y\,dx+(2-e^x)\sec^2 y\,dy=0$, $y=\pi/4$, $x=0$ $\tan y=2-e^x$

(II) $\frac{dy}{dx}+2y\tan x=\sin x$, $y=0$, $x=\pi/3$ $y\sec^2 x=\sec x-2$

P.T.O

(iii) $(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$ — $y = \tan^{-1}x - 1 + ce^{-\tan^{-1}x}$

(iv) $x^2 dy + (xy+y^2)dx = 0$, $y(1)=1$ — $y + 2x = 3x^2y$

(v) $\frac{dy}{dx} - \frac{y}{x} + \cos(\frac{y}{x}) = 0$, $y(1)=0$ — $\cos\frac{y}{x} = \log x + 1$

(vi) $\cos^2 x \frac{dy}{dx} + y - \tan x = 0$ — $y = \tan x - 1 + ce^{-\tan^{-1}x}$.

(vii) $(x^2-y^2)dx + 2xy\,dy = 0$, $y=1, x=1$ — $x^2+y^2 = 2x$

(viii) $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, $y=0, x=0$ — $2 - e^y = \frac{1}{x+1}$

(ix) $(\tan^{-1}y - x)dy = (1+y^2)dx$ — $x = \tan^{-1}y - 1 + ce^{\tan^{-1}y}$

(x) $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, $y(0)=1$ — $\log y = \frac{x^2}{2y^2}$

(xi) $(x-y)\frac{dy}{dx} = (x+2y)$, $y=0, x=1$ — $\begin{cases} -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right| + \\ \sqrt{3}\tan^{-1}\left(\frac{\frac{2y}{x}+1}{\sqrt{3}}\right) = \log|x| + \sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}} \end{cases}$

(xii) $\frac{dy}{dx} = 1+x+y+xy$, $y(1)=0$ — $\log(1+y) = \frac{x^2+2x-3}{2}$

(xiii) $\frac{dy}{dx} = \tan(x+y)$ — $y - x + \log|\cos(x+y) + \sin(x+y)| = c$

(xiv) $\sec^2 y(1+x^2)dy + 2x\tan y\,dx = 0$, $y=\frac{\pi}{4}$, $x=1$ — $(1+x^2)\tan y = 2$

(xv) $(x+y)^2\frac{dy}{dx} = 1$, $y=0$ $x=1$ — $y = \tan^{-1}(x+y) + \pi/4$

INTEGRATION

Q37 $\int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})\,dx$ — $\frac{\sqrt{2}\pi}{2}$

Q(38) $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$ — $\frac{\pi^2}{16}$

Q.(39) $\int \frac{dx}{\cos^4 x + \sin^4 x}$ — $\frac{1}{\sqrt{2}}\tan^{-1}(\tan x - \cot x) + c$

Q40 $\int_0^{\pi/2} (2\log(\sin x) - \log\sin 2x)\,dx$ — $\pi/2 \log\frac{1}{2}$

P.T.O

Q (41) $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$ $\frac{\pi^2}{2} - \pi$

Q.42 $\int_0^4 [\,|x-1| + |x-2| + |x-4|\,]\, dx$ $23/2$

Q.43 $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$ $a\pi$

Q.44 $\int \frac{dx}{\sin x + \sin 2x}$ ~~$\frac{3}{2}\tan^{-1} x$~~ $\frac{1}{6}\log|1-\cos x| + \frac{1}{2}\log|1+\cos x| - \frac{2}{3}\log|1+2\cos x| + c$

Q.45 $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$ π

Q.46 $\int_0^{\pi} \frac{x}{1+\cos\alpha \sin x}\, dx$ $\frac{\pi(\pi-\alpha)}{\sin\alpha}$

Q.47 $\int (x+3)\sqrt{3-4x-x^2}\, dx$ $-\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$

Q.48 $\int_0^{\pi} \frac{4x \sin x}{1+\cos^2 x}$ π^2

Q.49 $\int \frac{1-x^2}{x(1-2x)}\, dx$ $\frac{x}{2} + \log|x| - \frac{3}{4}\log|1-2x| + c$

Q.50 $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}\, dx$ $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c$

Q.51 $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx$ $-\pi/2 \log 2$

Q.52 $\int \frac{1}{(x-1)\sqrt{2x-3}}\, dx$ $\frac{2}{\sqrt{5}}\tan^{-1}\sqrt{\frac{2x-3}{5}}$

Q.53 $\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\, dx$ $x + \frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$

Q.54 $\int_0^1 (3x^2+2x+1)\, dx$ Limit as sum. (3)

Q.55 $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\, dx$ $x + c$

Q.56 $\int \frac{x^2}{x^4-x^2-12}\, dx$ $\frac{1}{7}\log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}} + c$

Q.57 $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ $\quad a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + c$

Q.58 $\int \sqrt{\tan x}\,dx$ $\quad \frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{\tan x - \sqrt{2\tan x + 1}}{\tan x + \sqrt{2\tan x + 1}}\right| + c$

Q.59 $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$, $\frac{\pi}{8}\log 2$

Q.60 $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$, $\frac{\pi}{2} - \log 2$

Q.61 $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$, $\frac{\pi}{2} - \log 2$

Q.62 $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$, π^2

Q.63 $\int_0^{\pi/2} \log(\tan x + \cot x)\,dx$, $\pi \log 2$

Q.64 $\int_0^3 (x^3 + e^{3x+1})\,dx$ Limit as sum. $\frac{81}{4} + \frac{e^{10} - e}{3}$

Q.65 $\int_{-2}^{2} \frac{x^2}{1+5^x}\,dx$, $8/3$

Q.66 $\int \frac{dx}{\sin(x+a)\sin(x-b)}$, $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x+a)}{\sin(x-b)}\right| + c$

Q.67 $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx$, $\frac{1}{40}\log 9$

Q.68 $\int_0^{\pi} \frac{x\,dx}{25\sin^2 x + 16\cos^2 x}$, $\frac{\pi^2}{20}$

Q.69. $\int_0^{\pi/2} \frac{\cos x}{1+\sin x+\cos x}\,dx$, $\frac{\pi}{4} - \frac{\log 2}{2}$

Q.70 $\int_0^{\pi} \frac{x\sin x}{1+3\cos^2 x}\,dx$, $\frac{\pi^2}{3\sqrt{3}}$

Q.71 $\int \frac{e^x}{(2+e^x)(4+e^{2x})}\,dx$ $\quad \frac{1}{8}\log|2+e^x| - \frac{1}{16}\log|4+e^{2x}| + \frac{1}{8}\tan^{-1}\frac{e^x}{2} + c$